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NONSTEADY ESCAPE OF GAS INTO A VACUUM THROUGH A SEMIPERMEABLE SCREEN

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UDC 533.6.011.533.697

The problem of the distribution of the parameters of a gas in a rarefaction wave during the nonsteady escape of the gas into a vacuum through a screen, which has a hydrodynamic resistance and removes part of the gas energy, is solved by the method of the theory of similarity and dimensionalities.

Suppose that a plane $x = 0$ (Fig. 1) separates a left-hand half-space $x < 0$, filled with an ideal gas having the parameters ρ_0, p_0, and T_0 and an equation of state $p = \rho T$, from a right-hand half-space, a vacuum 1 ($x > 0$).

At some moment the gas starts to escape into the vacuum through an infinitely thin screen 2 located in this plane which possesses a hydrodynamic resistance and removes part of the energy of the stream. The front of a rarefaction wave 3 propagates away from the screen to the left (through the undisturbed gas) and the boundary of the expanding gas 4 propagates to the right. The parameters of the flow in the rarefaction wave to the left of the plane $x = 0$ have the index 1 while the parameters of the flow to the right of this plane have the index 2.

We assume that the specific flow rate of gas through the screen depends on the pressure drop at the screen in the following way:

$$q = \alpha(p_{10} - p_{20}),$$

where p_{10} and p_{20} are the gas pressures at the plane $x = 0$ to the left and right of the screen; α is the coefficient of permeability of the screen.

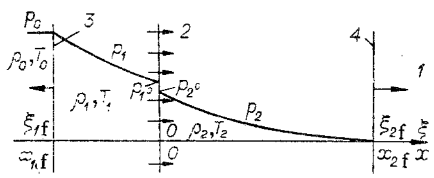

Fig. 1

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The equations of gas dynamics for the plane, one-dimensional, nonsteady flow are written in the form

$$\frac{\partial\rho}{\partial t}+v\frac{\partial\rho}{\partial x}+\rho\frac{\partial v}{\partial x}=0,\quad \frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{T}{\rho}\frac{\partial\rho}{\partial x}+\frac{\partial T}{\partial x}=0,\quad \frac{\partial T}{\partial t}+v\frac{\partial T}{\partial x}+(\gamma-1)T\frac{\partial v}{\partial x}=0, \tag{1}$$

where $\rho(x, t)$, $v(x, t)$, and $T(x, t)$ are the density, velocity, and temperature of the gas.

The parameters determining the process of escape will be the density and temperature of the undisturbed gas, ρ_0, kg/m^3, and T_0, m^2/sec^2, and the coefficient of permeability of the screen, α, sec/m, two of which (ρ_0 and T_0) have an independent dimensionality (the dimensionality of α can be expressed from the dimensionality of T_0). In this connection the problem under consideration belongs to the class of self-similar problems [2] with one independent variable

$$\xi = x/T_0^{1/2}t. \tag{2}$$

The density, velocity, and temperature distributions will be sought as functions of this variable:

$$\rho=\rho_0 f(\xi),\quad v=T_0^{1/2}\varphi(\xi),\quad T=T_0\psi(\xi). \tag{3}$$

By the substitution (3), with allowance for (2), we can change from the system of equations (1) to the system of ordinary differential equations

$$(\varphi-\xi)\frac{d\ln f}{d\xi}+\frac{d\varphi}{d\xi}=0,\quad (\varphi-\xi)\frac{d\varphi}{d\xi}+\frac{d\psi}{d\xi}+\psi\frac{d\ln f}{d\xi}=0,\quad (\varphi-\xi)\frac{d\psi}{d\xi}+(\gamma-1)\psi\frac{d\varphi}{d\xi}=0. \tag{4}$$

By expressing $d\ln f/d\xi$ from the first equation of the system (4) and $d\psi/d\xi$ from the third and substituting into the second, we can obtain the equation

$$\frac{d\varphi}{d\xi}\left[(\varphi-\xi)^2-\gamma\psi\right]=0,$$

whose nontrivial solution is

$$\psi=\frac{1}{\gamma}(\varphi-\xi)^2. \tag{5}$$

The substitution of (5) into the system (4) allows us to determine all the unknown functions:

$$f(\xi)=B\left(A-\frac{\gamma-1}{\gamma+1}\xi\right)^{\frac{2}{\gamma-1}},\quad \varphi(\xi)=\frac{2}{\gamma+1}\xi+A,\quad \psi(\xi)=\frac{1}{\gamma}\left(A-\xi\frac{\gamma-1}{\gamma+1}\right)^2, \tag{6}$$

where A and B are integration constants determined by the boundary conditions in the left-hand and right-hand half-spaces.

First let us consider the flow in the left-hand half-space:

$$x_{1f}\leqslant x\leqslant 0,\quad x_{1f}=\xi_{1f}T_0^{1/2}t,$$

where x_{1f} is the coordinate of the front of the rarefaction wave; ξ_{1f} is the constant of the front, also determined from the boundary conditions.

At the front $x = x_{1f}$, $\xi = \xi_{1f}$ of the rarefaction wave the gas density and temperature reach the values corresponding to the parameters of the undisturbed gas:

$$\rho(x_{1f},\, t)=\rho_0,\quad T(x_{1f},\, t)=T_0.$$

Here the gas velocity is equal to zero,

$$v(x_{1f},\, t)=0.$$

Consequently, with allowance for (6), we have the following boundary conditions for the left-hand half-space:

$$\frac{2}{\gamma+1}\xi_{1f}+A_1=0,\quad \frac{1}{\gamma}\left(A_1-\frac{\gamma-1}{\gamma+1}\xi_{1f}\right)^2=1,\quad B_1\left(A_1-\frac{\gamma-1}{\gamma+1}\xi_{1f}\right)^{\frac{2}{\gamma-1}}=1. \tag{7}$$

Solving the system (7), we obtain the values of the unknown constants:

$$\xi_{1f}=-\gamma^{1/2},\quad A_1=\frac{2\gamma^{1/2}}{\gamma+1},\quad B_1=\gamma^{-\frac{1}{\gamma-1}}. \tag{8}$$

Thus, with allowance for (3), (5), and (8) the unknown dimensionless distributions of density, velocity, and temperature for the left-hand half-space are obtained from (6) by a substitution of the constants A_1 and B_1 of (8).

Now we can determine the gas pressure to the left of the screen ($x = 0$, $\xi = 0$)

$$p_{10} = (\rho_1 T_1)|_{x=0} = \rho_0 T_0 \left(\frac{2}{\gamma+1}\right)^{\frac{2\gamma}{\gamma-1}} \tag{9}$$

and the gas flow rate through the screen,

$$q = (\rho_1 v_1)|_{x=0} = \rho_0 T_0^{1/2} \gamma^{1/2} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}. \tag{10}$$

Let us consider the gas flow in the right-hand half-space

$$0 \leqslant x \leqslant x_{2f} = \xi_{2f} T_0^{1/2} t,$$

where x_{2f} is the coordinate of the boundary of propagation of the gas; ξ_{2f} is the constant for this boundary. At $x = x_{2f}$ the gas density and temperature are equal to zero. Consequently, allowing for (6), we have

$$A_2 - \frac{\gamma-1}{\gamma+1} \xi_{2f} = 0. \tag{11}$$

The gas flow rate through the screen is

$$q = (\rho_2 v_2)|_{x=0} = \rho_0 T_0^{1/2} B_2 A_2^{\frac{\gamma+1}{\gamma-1}}. \tag{12}$$

From the law of conservation of mass (10) and (12) must be equal, from which we get

$$B_2 A_2^{\frac{\gamma+1}{\gamma-1}} = \gamma^{1/2} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}. \tag{13}$$

The gas pressure to the right of the screen is

$$p_{20} = p_{10} - q/\alpha.$$

Consequently, with allowance for (9) we have

$$B_2 A_2^{\frac{2\gamma}{\gamma-1}} = \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{2\gamma}{\gamma-1}} - \frac{\gamma^{3/2}}{\alpha T_0^{1/2}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}. \tag{14}$$

Thus, the unknown constants A_2, B_2, and ξ_{2f} can be determined from the boundary conditions (11), (13), and (14):

$$A_2 = \gamma^{1/2}\left(\frac{2}{\gamma+1}\right) - \frac{\gamma}{\alpha T_0^{1/2}}\left(\frac{\gamma+1}{\gamma-1}\right), \quad B_2 = \frac{\gamma^{1/2}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}{\left[\gamma^{1/2}\left(\frac{2}{\gamma+1}\right) - \frac{\gamma}{\alpha T_0^{1/2}}\right]^{\frac{\gamma+1}{\gamma-1}}},$$
$$\xi_{2f} = \frac{2\gamma^{1/2}}{\gamma-1} - \frac{\gamma}{\alpha T_0^{1/2}}\left(\frac{\gamma+1}{\gamma-1}\right). \tag{15}$$

The unknown distributions of the dimensionless densities, velocities, and temperatures are obtained by substituting A_2 and B_2 of (15) into (6).

Finally, the distributions of the gas densities, velocities, and temperatures can be represented in the form (see following page)

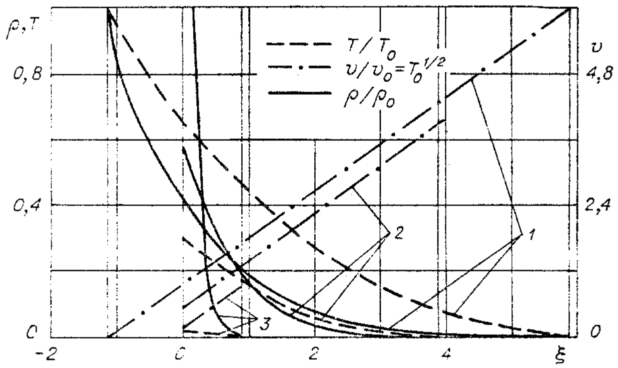

Fig. 2

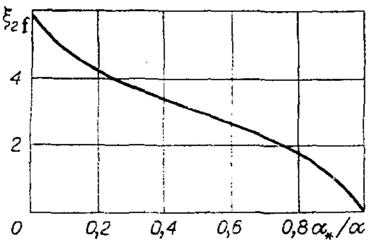

Fig. 3

$$\left.\begin{aligned} \rho &= \rho_0 \gamma^{-\frac{1}{\gamma-1}} \left(\frac{2\gamma^{1/2}}{\gamma+1} - \frac{\gamma-1}{\gamma+1}\xi\right)^{\frac{2}{\gamma-1}}, \\ v &= T_0^{1/2}\left(\frac{2}{\gamma+1}\xi + \frac{2\gamma^{1/2}}{\gamma+1}\right), \\ T &= T_0 \frac{1}{\gamma}\left(\frac{2\gamma^{1/2}}{\gamma+1} - \frac{\gamma-1}{\gamma+1}\xi\right)^2, \end{aligned}\right\} -\gamma^{1/2} \leqslant \xi \leqslant 0,$$

$$\left.\begin{aligned} \rho &= \rho_0 \frac{\gamma^{1/2}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}{\left(\gamma^{1/2}\frac{2}{\gamma+1} - \frac{\gamma}{\alpha T_0^{1/2}}\right)^{\frac{\gamma+1}{\gamma-1}}} \\ &\times \left(\frac{2\gamma^{1/2}}{\gamma+1} - \frac{\gamma}{\alpha T_0^{1/2}} + \frac{\gamma-1}{\gamma+1}\xi\right)^{\frac{2}{\gamma-1}}, \\ v &= T_0^{1/2}\left(\frac{2}{\gamma+1}\xi + \frac{2\gamma^{1/2}}{\gamma+1} - \frac{\gamma}{\alpha T_0^{1/2}}\right), \\ T &= T_0 \frac{1}{\gamma}\left(\gamma^{1/2}\frac{2}{\gamma+1} - \frac{\gamma}{\alpha T_0^{1/2}} + \frac{\gamma-1}{\gamma+1}\xi\right)^2, \end{aligned}\right\} 0 \leqslant \xi \leqslant \frac{2\gamma^{1/2}}{\gamma+1} - \frac{\gamma(\gamma+1)}{\alpha T_0^{1/2}(\gamma-1)}.$$

Distributions of the main parameters of the flow for $\gamma = 1.4$ and different values of the hydrodynamic resistance, $\Delta p = p_{10} - p_{20}$, and the energy removed by the screen,

$$\Delta w = \rho_{10} v_{10}\left(\frac{v_{10}^2}{2} + \gamma\frac{T_{10}}{\gamma-1}\right) - \rho_{20} v_{20}\left(\frac{v_{20}^2}{2} + \gamma\frac{T_{20}}{\gamma-1}\right),$$

normalized to the values of the parameters p_0 and w_0 of the undisturbed stream, are given in Fig. 2: 1) $\alpha = \infty$, $\Delta w = \Delta p = 0$; 2) $\alpha = 3\alpha_*$, $\Delta w/w_0 = 0.38$, $\Delta p/p_0 = 0.1$; 3) $\alpha = 1.16\alpha_*$, $\Delta w/w_0 = 0.96$, $\Delta p/p_0 = 0.24$.

As seen from Fig. 2, when $\Delta p/p_0 = 0$ and $\Delta w/w_0$, which corresponds to total permeability of the screen ($\alpha = \infty$), the gas flow has the character of an ordinary centered rarefaction wave [1]. A decrease in α (an increase in Δp and Δw) leads to a decrease in the temperature, velocity, and constant ξ_{2f} of the gas front to the right of the screen and an increase in the gas density. At some limiting value of $\alpha_* = (\gamma + 1)\gamma^{1/2}/2T_0^{1/2}$, $\Delta w/w_0 = 1$, and all the energy of the gas is absorbed by the screen. In this case the velocity, temperature, and constant of the front of the gas passing through the screen are equal to zero and the gas density approaches infinity, i.e., all the gas remains in the screen.

We note that a change in α does not affect the flow in the left-hand half-space, since at the plane $x = 0$ the gas has a velocity equal to the local velocity of sound, and disturbances from the screen can only propagate into the region of the right-hand half-space.

The velocity

$$\frac{dx_f}{dt} = \frac{d}{dt}(\xi_f T_0^{1/2} t) = \xi_f T_0^{1/2}$$

of motion of the front is determined by the value of the constant ξ_f of the front. The dependence of the constant ξ_{2f} of the boundary of a gas propagating into a vacuum on the parameter α_*/α is presented in Fig. 3.

The velocity of the front of the rarefaction wave is equal to the velocity of sound in the undisturbed gas:

$$\xi_{1f} T_0^{1/2} = (\gamma T_0)^{1/2}.$$

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TEMPERATURES OF THE IMPULSE COMPRESSION OF IONIC CRYSTALS

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UDC 539.21

In [1, 2] a nonparametric calculation of impulse adiabatics of ionic crystals in B1 and B2 phases was given. The relations $p_H = p_H(V)$ obtained enable the temperature of the impulse compression T_H to be obtained for the crystals considered in both phases and enable the effect of the B1 → B2 phase transition on the $T_H = T_H(p_H)$ curves to be investigated.

Taking the energy of the thermal vibrations in the form $C_V T$ with constant heat capacity C_V and writing the internal energy using the impulse adiabatic equation we obtain the following equation for the temperature of the impulse compression [3]:

$$T_H = \left[\frac{1}{2} p_H(V)(V_0 - V) + E_0 - E_x(V)\right] \Big/ C_V, \tag{1}$$

TABLE 1

Crystal	B1 phase		B2 phase	
	p_H, kbar	T_H, °K	p_H, kbar	T_H, °K
LiF	64	319	548	527
	175	366	941	1490
	335	502	1550	3430
	569	792	2540	7330
	915	1420		
NaF	54	329	199	374
	115	389	352	998
	202	505	582	2 130
	325	788	937	4 150
	501	1340	1503	7 880
NaCl	12	308	108	533
	31	337	171	100
	56	384	263	1 850
	91	473	393	3 140
	136	643	587	5 380
	198	944	886	9 140
	282	1520	1370	15 940
KCl	27	313	56	362
	50	343	99	715
	80	452	163	1 370
	120	627	255	2 400
	176	970	394	4 280
	253	1630	614	7 430
	362	2650	979	13 280
KBr	26	317	51	435
	44	351	89	803
	69	444	142	1 430
	101	614	219	2 460
	145	989	333	3 396
	206	1460	511	7 322

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